Multiple partition curve analysis to estimate sampling induced uncertainty

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Partition Curves

• Partition curves are used in coal and mineral processing (and can be used in many other industries) to determine process efficiency.

• A Partition curve shows the amount of material, with a particular quality, that appeared in the product (or reject) compared to how much of that same material was in the feed.

• These material qualities may be based on particle size, density or other material variables.
Partition Curve
Partition Curve Example

• If the feed to a density separation process, contains 200 t/h of 1.45 to 1.50 t/m^3 material.

• If the product contains 182 t/h of 1.45 to 1.50 t/m^3 solids.

• The partition coefficient for 1.45 to 1.50 t/m^3 material is 182/200 = 0.91 or 91 % partition coefficient to product.

• Partition coefficients can be expressed as either to product or to reject and are simply 100 minus the other.
Feed

• 200 t/h
• 1.45 to 1.5 t/m^3

Process

• Partition Coefficient to Product of 1.45 to 1.5 t/m^3 material = 91 %

Product

• 182 t/h
• 1.45 to 1.5 t/m^3

Reject
Partition Curve
• Once the Partition Curve has been constructed for all density or size intervals, then the cutpoint and Ep can be read from the curve.

• The cutpoint is the separation point and the Ep is sharpness of separation (slope of the curve). Defined as \((D75 - D25)/2\)
• D50 = 1.53 t/m^3
• Ep = (d75 – d25) / 2
• D75 = 1.51. D25 = 1.56. Ep = 0.025
• If the slope were vertical then perfect separation would be achieved and the Ep would be 0. As the curve becomes more horizontal, then separation efficiency is worse and the Ep value increases.
• From this curve can also be determined the amount of misplaced material, that is, the amount of material that ideally should have been in product and ended up and reject and vice versa.
Partition Curve Construction

• To determine the flowrates of each density or size fraction, the total flowrates are found for each stream and then each stream is analysed for the mass of material in each relevant density or size fraction.
Determining Flowrates

• From ISO 923, the flowrates of each stream should be found by actually measuring these flowrates.

• If these flowrates cannot be measured directly, then the relative flowrates are to be found by sample analysis and mass balancing, but this is to be used only as a last resort.

• In practice it is very difficult to measure the actual mass flowrates and mass balancing is used as the first means.
Example in Paper – Hydrocyclone sizing operation

- Mass Yield = Product flowrate / Feed flowrate
- MYa (Mass Yield Calculated from ash)
- Feed Head Ash 13.5 %,d
- Overflow Head Ash 10.3 %,d
- Underflow Head Ash 16.7 %,d

- MYa = \frac{(t-f)}{(t-p)} * 100
- = 50 %,d/d
Partition Curve Construction

• The Standard method (ISO 923 and AS 4156.7) is to use the Product and Reject stream data, in combination with the mass yield, to create a Reconstituted feed and then plot the Product or Reject over that Reconstituted Feed to produce the Partition Curve and to do this for only one value of MY.

Normal to plot Actual Product / Reconstituted Feed

• This gives numbers that are in most cases believable and are usually given without any uncertainty.
Standard Partition Curve
• MYa = 50 % d/d
• d50 = 0.3 mm
• d75 = 0.41 mm
• d25 = 0.21 mm
• Ep = (0.41 - 0.21)/2 = 0.1

• Everyone is happy, BUT
Mass Balancing Formula

- $MYsc = \frac{t-f}{t-p} \times \frac{p}{f} \times 100$
- $MYrho = \frac{t-f}{t-p} \times 100$
- $MYa = \frac{t-f}{t-p} \times 100$
- and others
- Differences between these give the first set of uncertainty.
MYa

- MYa is the most commonly used in coal preparation and is analogous to elemental content in minerals processing,
- But it is not as accurate as ash is not present in coal, mineral matter is.
- The conversion of mineral matter to ash is not always constant due to reactions between pyrite and calcite.
- Work is continuing on this (myself and Michael Campbell) and will be presented at the 14th ACPS conference.
- However, that is separate to the point of this presentation.
Mass Balancing Formula

• MYsc = \( \frac{(t-f)}{(t-p)} \times \frac{p}{f} \times 100 \)

• MYrho = \( \frac{(t-f)}{(t-p)} \times 100 \)

• MYa = \( \frac{(t-f)}{(t-p)} \times 100 \)

• Differences between these give the first set of uncertainty.
• Normal to plot Actual Product / Reconstituted Feed

• Of course, the Partition Curve could be plotted with Actual Product over Actual Feed, Reconstituted Product (derived from Feed and Reject and MY), over Actual Feed and so on.

• Actual Product / Reconstituted Feed
• Actual Product / Actual Feed
• Reconstituted Product/ Reconstituted Feed
• Reconstituted Product/ Actual Feed
• And the same for Reject

• This leads to eight curves that can be plotted to give the partition.
• All eight curves for each Mass Yield can be plotted and all the various values of D50 and Ep can be found, the values tabulated and the MINIMUM uncertainty found.
The reason that Actual Product / Reconstituted Feed is used is that it is thought that particle degradation may occur during the process. Due to size degradation that can occur during the process, it is common to use only the actual product, actual reject and reconstituted feed. However, in many cases we have found that this size degradation does not occur or the sampling error is greater than the size degradation. In the example given in our paper, it was found that the actual feed was finer than the reconstituted feeds.
Particle Size Degradation
• Performance Indicators
  
  **MY to U/F**

  - 48 % d/d ± 13 % d/d (abs)
  - 35 % d/d to 61 % d/d

  d50 0.32 mm ± 0.04 mm

• Ep 0.20 ± 0.13 0.07 to 0.33

• The exercise was to find the efficiency of separation of the hydrocyclone. Complete waste of time and money
• This method is not yet definitive, it is simply the basis by which a start can be made at using partition curves and yields to give a simple, quick estimate of sampling uncertainty.

• If large error exists, then this type of work can be used to justify the expense of improving the sampling systems.
• The error shown here is the total error of sampling and analyses.

• It is generally accepted within the coal preparation industry within Australia that 85 % of error is with the sampling, 10 % with sample preparation and 5 % with sample analysis (The Principles of Coal Preparation, Australian Coal Preparation Society)
In the Australian Coal Preparation Industry, we currently use large amounts of data regression to give partition curves and yields that balance perfectly. It is the authors’ opinion that we should be using the data (and methods such as multiple partition curve analysis) to find the error in sampling and then attempt to reduce that error.
Solving the Problem

• Increased awareness and revised methodology is vitally important for improving industry standards. In plants we can get the sampling right, we have access to the total flow of material. We just need to plan for it in the construction of the plants.

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